

Casimir torque

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Abstract. We develop a formalism for the calculation of the flow of angular momentum carried by the fluctuating electromagnetic field within a cavity bounded by two flat anisotropic materials. By generalizing a procedure employed recently for the calculation of the Casimir force between arbitrary materials, we obtain an expression for the torque between anisotropic plates in terms of their reflection amplitude matrices. We evaluate the torque in 1D for ideal and realistic model materials.

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1. Introduction

In the last decade the Casimir effect [1] has received considerable attention, as the recently attained high experimental accuracy has permitted detailed tests of theoretical predictions [2, 3, 4, 5, 6, 7, 8, 9]. This, in turn, has stimulated a growing interest in fundamental aspects of the vacuum field. The study of vacuum forces between realistic materials was pioneered by Lifshitz [10], who considered local homogeneous materials whose fluctuating currents were the sources of the fluctuating electromagnetic field and whose correlations were related to the dielectric response of the materials. One of the limitations of the Lifshitz theory is the requirement of a definite microscopic model of matter which has to be solved simultaneously with the electromagnetic field equations. Thus, the applicability of the results seem to be limited by the generality of that initial model. In particular, Lifshitz results were developed for isotropic materials. Nevertheless, the vacuum energy has been calculated for cavities bounded by anisotropic materials, first in the non-retarded limit [11] and later for arbitrary distances [12], resulting in a torque whenever the optical axes of the plates are not aligned with each other. An alternative derivation of the Casimir torque in the 1D case has been developed [13] starting from the angular momentum flux carried by the field. Analytical formulae have been found in the retarded limit when the anisotropy is small [14]. Numerical calculations have also been performed for materials with a small anisotropy and it has been shown that the torque may be large

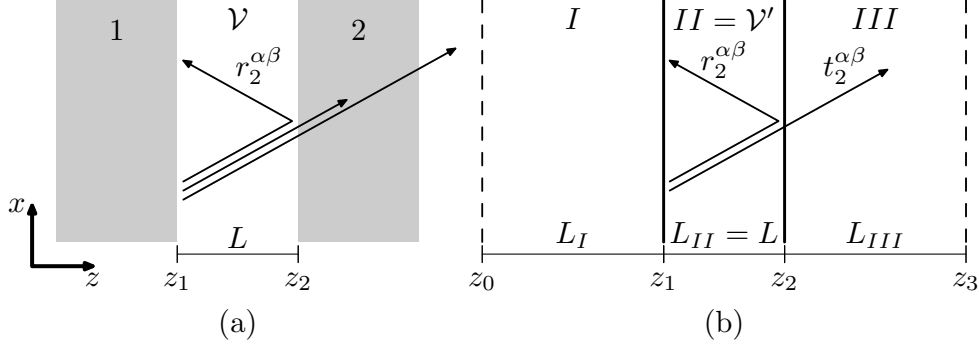


Figure 1. (a) Vacuum cavity \mathcal{V} of width L bounded by two arbitrary material slabs (1 and 2) with surfaces at z_1 and z_2 . A photon with polarization \hat{e}_i^β propagating within the cavity is either reflected coherently with amplitude and polarization $r_2 \hat{e}_r^\alpha = r_2^{\alpha\beta} \hat{e}^\beta$, or lost from the cavity with probability $1 - |r|^2$. (b) Fictitious system made up three empty regions I , II , and III , bounded by perfect mirrors at z_0 and z_3 and with infinitely thin sheets at z_1 and z_2 whose reflection amplitudes $r_a^{\alpha\beta}$ are identical to those of the real system and whose transmission amplitudes $t_a^{\alpha\beta}$ are such that energy is conserved with no absorption whatsoever. The field within the fictitious cavity \mathcal{V}' (region II) is the same as within the real cavity \mathcal{V} .

enough to be experimentally measurable in several novel experimental configurations [15].

In the previous theoretical works essential assumptions about the dielectric properties of the plates were done from the onset in order to derive expressions for the Casimir torque. However, recent works [16, 17] have shown that if the theory is set up in terms of the reflection coefficients of the media, it is possible to decouple the calculation of the Casimir force from the calculation of the dielectric response of the materials. The so called scattering approach has permitted the calculation of the force for a wide class of systems simply by plugging into the resulting formulae the appropriate reflection amplitudes or surface impedances. Thus, transparent and opaque, local and non-local, infinite and finite, homogeneous and heterogeneous systems may be treated in the same footing. In the present paper we generalize the scattering approach to account for anisotropy as well. We present a new derivation of the Casimir torque between plates with arbitrary dielectric properties characterized by their anisotropic optical coefficients. For simplicity, in this paper we focus our attention on one dimensional systems, although our approach is also applicable to 3D [18]. We present results for both ideal systems, for which analytical formulae are obtained, and realistic dichroic systems.

2. Scattering approach

To calculate the torque we follow the scattering approach [16, 17], illustrated by Fig. 1(a). A photon within the cavity may be described by its amplitude \mathcal{E} , frequency ω , wavevector $\vec{k} = (\vec{Q}, \pm q)$ and polarization \hat{e}^α . We may choose the independent polarizations as $\alpha = s, p$ (or equivalently, TE and TM). When a photon with polarization \hat{e}_i^α is incident upon the surface, say, of medium 2 at z_2 , it is either reflected coherently with an amplitude and polarization proportional to $r_2 \hat{e}_r^\alpha \equiv r_2^{\alpha\beta} \hat{e}_i^\beta$

(sum implied), or transmitted with a probability $1 - |r_2|^2$. Here, $r_a^{\alpha\beta}$ is a 2×2 matrix that describes the reflection amplitude of medium $a = 1, 2$. For isotropic media, $r_a^{\alpha\beta}$ may be taken as a scalar whenever the incoming field has s or p polarization, but that separation is not possible in general when the media are anisotropic. We remark that $r_a^{\alpha\beta}$ is defined to be the complete reflection amplitude of medium a , not only that of its front surface. For instance, if medium a were a thin film or a layered system, the multiple reflections within a are to be incorporated into $r_a^{\alpha\beta}$. Thus, if the photon is not coherently reflected, it must necessarily be absorbed within a or else be transmitted into the empty space beyond. The principle of detailed balance implies that in thermodynamic equilibrium, for every photon that is not reflected and is therefore lost from the cavity either through absorption or transmission, an equivalent photon is incoherently launched into the cavity, either being radiated by the absorbing medium, or else, arriving from the vacuum beyond and being transmitted into the cavity. In any case, the probability that a photon with wavevector $(\vec{Q}, -q)$ and polarization \hat{e}_r^α arrives into the cavity from medium 2 with no phase relation to the lost photon is proportional to $1 - |r_2|^2$. Similar statements apply to medium 1.

From the previous discussion, it follows that in equilibrium the properties of the radiation field within the cavity \mathcal{V} depend on the cavity walls only through their optical coefficients $r_a^{\alpha\beta}$. Equivalently, the cavity radiation is completely determined by the exact surface impedance $Z_a^{\mu\nu}$ defined through $[\hat{n}_a \times (\hat{n}_a \times \vec{E}_a)]^\mu = Z_a^{\mu\nu} (\hat{n}_a \times \vec{H}_a)^\nu$ (sum implied), where \hat{n}_a denotes the outgoing unit normal of surface a , \vec{E}_a and \vec{H}_a are the total electric and magnetic fields at z_a and $\mu, \nu = x, y$ denote Cartesian coordinates along the walls. Thus, the electromagnetic radiation within the real cavity \mathcal{V} would be identical to that within a fictitious cavity \mathcal{V}' bounded by infinitely thin sheets at z_1 and z_2 , provided their reflection amplitudes $r_a^{\alpha\beta}$ are chosen to be equal to those of the walls of \mathcal{V} . Their transmission amplitudes $t_a^{\alpha\beta}$ may then be chosen in order to guarantee energy conservation with no absorption whatsoever of electromagnetic energy (Fig. 1(b)). As there is no absorption in the fictitious system, there is no excitation of material degrees of freedom and the normal modes of the electromagnetic field form a complete orthogonal basis of the corresponding Hilbert space, allowing the use of well developed quantum-mechanical procedures for the calculation of the field properties. Contrariwise, in the real system the electromagnetic energy is absorbed, probably exciting electronic or vibrational transitions, so that the problem cannot be treated quantum mechanically without incorporating the electronic and/or vibrational degrees of freedom into the calculation, which would require in turn the use of a microscopic model of the material.

In Fig. 1(b) we have included two perfect mirrors at positions z_0 and z_3 in order to quantize and count the normal modes of the system. The photons that are reflected at z_0 and z_3 and are transmitted back into \mathcal{V}' mimic the photons injected into the real cavity \mathcal{V} in order to restore thermal equilibrium, and in the limit $L_I, L_{III} \rightarrow \infty$ their phase is so large and so rapidly varying with ω that it effectively bears no relation with the phase of the photons lost from the cavity.

Using the scattering approach we can treat dissipationless, homogeneous, isotropic, local, sharp media on the same footing as dissipative, inhomogeneous (layered systems, superlattices, photonic structures), chiral, spatially dispersive materials with a smooth selvedge. In particular, we can treat anisotropic systems.

3. Torque in 1D

We consider a finite beam propagating within \mathcal{V}' along $\pm z$,

$$\vec{E}(\vec{r}, t) = E_r \hat{e}_r e^{i(qz - \omega t)} + E_l \hat{e}_l e^{-i(qz + \omega t)}, \quad (1)$$

where the subindices r and l denote right and left moving contributions, E_r and E_l are the corresponding amplitudes which we take as slowly varying functions of \vec{r} , and \hat{e}_r and \hat{e}_l the polarizations within the $x - y$ plane. To ensure that the field is divergence-less, an additional field contribution along z has to be added to (1),

$$\Delta \vec{E}(\vec{r}, t) = \left(\hat{e}_r \cdot \nabla E_r e^{i(qz - \omega t)} - \hat{e}_l \cdot \nabla E_l e^{-i(qz + \omega t)} \right) i\hat{z}/q. \quad (2)$$

Expressions similar to (1) and (2) may also be written for the magnetic field. The torque τ_z over medium 2 may be obtained by integrating the angular momentum flux $\mathbf{M} = \vec{r} \times \mathbf{T}$ over a surface that surrounds it, where \mathbf{T} is the electromagnetic stress tensor. Thus,

$$\tau_z = -\frac{1}{8\pi} \text{Re} \int da [(\vec{r} \times \vec{E}^*)_z E_z + (\vec{r} \times \vec{B}^*)_z B_z], \quad (3)$$

where the integral is over the cross section of the beam. Notice that had we started our calculation with an infinitely extended plane wave, E_z and B_z would have been zero, but the integral in Eq. (3) would have been over an infinitely extended surface, yielding an ill defined result. On the other hand, starting from Eq. (3) we can take the limit of a plane wave, obtaining well defined expressions. The electromagnetic torque may be considered an edge effect that survives in the plane wave limit. Substituting (1) and (2) and similar expressions for the magnetic field \vec{B} , and after some manipulation, we obtain

$$\tau_z = -\frac{A}{8\pi q^2} \text{Re}(\vec{E} \times \partial_z \vec{E}^*)_z, \quad (4)$$

where $A \rightarrow \infty$ is the cross sectional area of the wavefront. It can easily be checked that Eq. (4) is consistent with the quantum mechanical view that each photon of energy $\hbar\omega$ carries an angular momentum $\pm\hbar$ along $\pm z$ with speed c , according to its helicity.

Now we consider one normal mode of the fictitious system with amplitude \mathcal{E}_0 and frequency ω , $\vec{E} = \mathcal{E}_0 \vec{\phi}(z) e^{-i\omega t}$, where

$$\vec{\phi}(z) = \vec{C}^\Lambda e^{iqz} + \vec{D}^\Lambda e^{-iqz}, \quad (\Lambda = I, II, III) \quad (5)$$

is a spinorial normalized *wavefunction* with components ϕ_μ ($\mu = x, y$), \vec{C}^Λ , \vec{D}^Λ are distinct coefficients within each region Λ and $q = \omega/c$. In the limit $L_I, L_{III} \rightarrow \infty$, the electromagnetic energy $U = [L_I(\|C^I\|^2 + \|D^I\|^2) + L_{III}(\|C^{III}\|^2 + \|D^{III}\|^2)]|\mathcal{E}_0|^2 A/8\pi$ and the normalization condition $1 = [L_I(\|C^I\|^2 + \|D^I\|^2) + L_{III}(\|C^{III}\|^2 + \|D^{III}\|^2)]$ are dominated by the large fictitious regions I and III, so we may identify $U = A|\mathcal{E}_0|^2/8\pi$ and solve for the amplitude $|\mathcal{E}_0|^2 = 8\pi f_\omega \hbar\omega/A$ in terms of the equilibrium photon occupation number $f_\omega = \coth(\beta\hbar\omega/2)/2$ at temperature $k_B T = 1/\beta$. Thus, the contribution to the torque (3) of one mode may be written as

$$\tau_z = -\frac{\hbar c}{2q} f_\omega [\phi_x \partial_z \phi_y^* - \phi_y \partial_z \phi_x^* + (\partial_z \phi_y) \phi_x^* - (\partial_z \phi_x) \phi_y^*]. \quad (6)$$

Now we label each mode by an index n and sum (6) over n to obtain the total torque

$$\tau_z = -\hbar c \int dq f_{qc} \sum_n \delta(q^2 - q_n^2) \times [\phi_{nx} \partial_z \phi_{ny}^* - \phi_{ny} \partial_z \phi_{nx}^* + (\partial_z \phi_{ny}) \phi_{nx}^* - (\partial_z \phi_{nx}) \phi_{ny}^*], \quad (7)$$

where we introduced the q integration and the Dirac's δ in order to write the result in terms of the Green's function $G_{\mu\nu}(z, z') = \sum_n \phi_{n\mu}(z) \phi_{n\nu}^*(z') / (q^2 + i\eta - q_n^2)$,

$$\tau_z = \frac{\hbar c}{\pi} \int_0^\infty dq f_{qc} (\partial_{z'} - \partial_z) [A_{xy}(z, z') - A_{yx}(z, z')]_{z' \rightarrow z}, \quad (8)$$

where $A_{\mu\nu}(z, z') = [G_{\mu\nu}(z, z') - G_{\nu\mu}(z', z)] / 2i$ is the anti-Hermitian part of $G_{\mu\nu}$. Here, η is a positive infinitesimal and we employed the identity $\pi\delta(x) = -\text{Im}(x + i\eta)^{-1}$. The Green's function may be evaluated by solving $(\partial_z^2 + q^2 + i\eta)G_{\mu\nu}(z, z') = \delta(z - z')\delta_{\mu\nu}$, subject to the appropriate boundary conditions. We write the solution in terms of the two homogeneous solutions $\vec{u}_\lambda(z)$ and $\vec{v}_\lambda(z)$ ($\lambda = 1, 2$) that satisfy the boundary conditions on the right and the left side of the system respectively,

$$\begin{aligned} \mathbf{G}(z, z') &= \mathbf{u}(z) [\mathbf{u}'(z) - \mathbf{v}'(z') \mathbf{v}^{-1}(z') \mathbf{u}(z')]^{-1} \theta(z - z') \\ &\quad - \mathbf{v}(z) [\mathbf{v}'(z') - \mathbf{u}'(z') \mathbf{u}^{-1}(z') \mathbf{v}(z')]^{-1} \theta(z' - z), \end{aligned} \quad (9)$$

where $\mathbf{G}(z, z')$ is a matrix with elements $G_{\mu\nu}(z, z')$, $\mathbf{u}(z)$ and $\mathbf{v}(z)$ are matrices with columns $\vec{u}_\lambda(z)$ and $\vec{v}_\lambda(z)$ respectively, i.e., with matrix elements $u_{\mu\lambda}(z)$ and $v_{\mu\lambda}(z)$, \mathbf{u}' and \mathbf{v}' denote their derivatives with respect to their argument, and θ denotes the Heaviside unit step function.

For the case of uniaxial or orthorhombic slabs with normal-incidence reflection amplitudes r_{x_a} and r_{y_a} in the principal axes x_a and y_a of the a -th slab we may write

$$\mathbf{u}(z) = \mathbf{R} \cdot \mathbf{u}_0(z), \quad \mathbf{v}(z) = \mathbf{R}^T \cdot \mathbf{v}_0(z), \quad (10)$$

where \mathbf{R} and \mathbf{R}^T are rotation matrices by angles $\gamma/2$ and $-\gamma/2$ respectively and

$$\mathbf{u}_0(z) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{iq(z-z_2)} + \begin{pmatrix} r_{x_2} & 0 \\ 0 & r_{y_2} \end{pmatrix} e^{-iq(z-z_2)}, \quad (11)$$

$$\mathbf{v}_0(z) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{-iq(z-z_1)} + \begin{pmatrix} r_{x_1} & 0 \\ 0 & r_{y_1} \end{pmatrix} e^{iq(z-z_1)}, \quad (12)$$

are the solutions at the right and left sides of \mathcal{V}' referred to the principal axes of the corresponding slab, which we assumed to be rotated by an angle γ with respect to those of the opposite slab. Notice that \mathbf{R} and \mathbf{R}^T act only on the first index μ of \mathbf{u} and \mathbf{v} .

Substituting Eqs. (10)-(12) in Eqs. (9) and (8) we obtain a simple expression

$$\tau_z = -\frac{\hbar c}{2\pi} \int_0^\infty d\kappa \frac{\Delta r_1 \Delta r_2 \sin 2\gamma e^{-2\kappa L}}{\Delta r_1 \Delta r_2 \sin^2 \gamma e^{-2\kappa L} + (1 - r_{1x} r_{2x} e^{-2\kappa L})(1 - r_{1y} r_{2y} e^{-2\kappa L})}, \quad (13)$$

where we also took the zero temperature limit $f_{qc} = 1/2$ and we deformed the q integration path from the positive real axis toward the positive imaginary axis, as is usual. Here, $\kappa = q/i$ and we defined the anisotropy $\Delta r_a = r_{x_a} - r_{y_a}$ of the a -th slab. Eq. (13) is the main result of the present paper. We have verified that it is equivalent to the result of Ref. [13], although in a much more compact form.

4. Results

As a first application of our result (13) we calculate the torque between two ideal perfectly reflecting mirrors covered by ideal perfectly absorbing polarizers, that is, we take $r_{x_1} = r_{x_2} = \pm 1$, $r_{y_1} = r_{y_2} = 0$. In this case, Eq. (13) may be integrated analytically, yielding

$$\tau_z = \frac{\hbar c}{2\pi L} \tan \gamma \log \sin^2 \gamma. \quad (14)$$

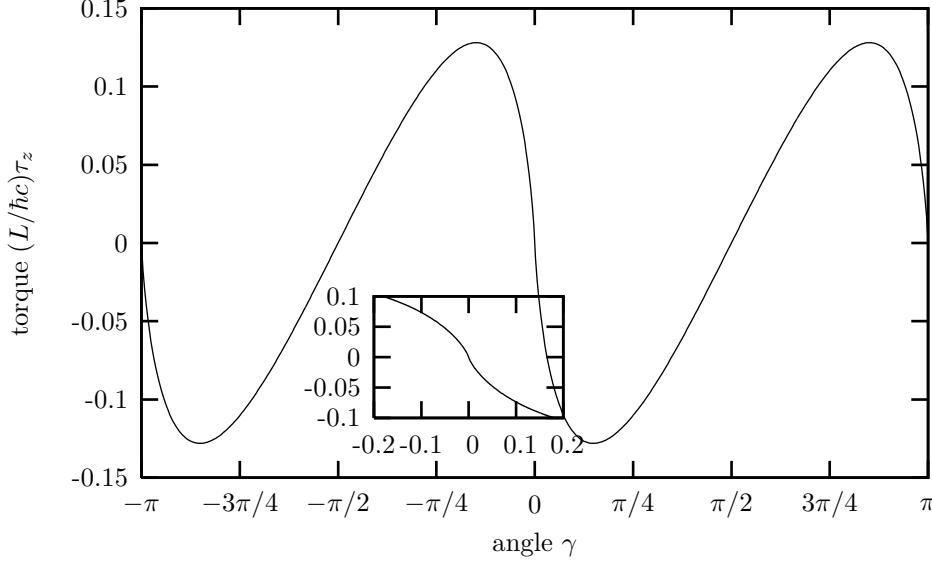


Figure 2. Torque in 1D at $T = 0$ between perfect mirrors covered by perfect polarizers as a function of the angle γ between corresponding principal directions.

Notice that the torque decays as $1/L$, in analogy to the $1/L^2$ decay of the Casimir force between perfect mirrors in 1D and the corresponding $1/L^4$ decay in 3D. In Fig. 2 we show the torque as a function of the angle. As could have been expected, it is a periodic function of γ with period π . It is null when both polarizers are aligned, $\gamma = 0$, corresponding to a stable equilibrium orientation, and when they are orthogonal to each other, $\gamma = \pm\pi/2$, corresponding to unstable equilibrium. The inset shows that the slope of $\tau_z(\gamma)$ is singular at the stable equilibrium point. We remark that the torque is not simply proportional to $\sin 2\gamma$ and therefore its extreme values are not at $\gamma = \pm\pi/4$. However, it is odd-symmetric around $\gamma = 0$. The maximum torque may be estimated as $0.1\hbar c/L$; for example, at $L = 10\text{nm}$ it is about 10^{-19}Nm . For dimensional reasons, in a full 3D calculation our result above would have to be scaled by A/L^2 multiplied by some dimensionless factor.

If we replace the perfect mirrors above by lossy mirrors with reflection amplitude r , we can again obtain an analytical expression

$$\tau_z = \frac{\hbar c \tan \gamma}{2\pi L} \log(1 - |r|^2 \cos^2 \gamma), \quad (15)$$

Fig. 3 shows that as $|r|$ diminishes τ_z is reduced and becomes closer to a simple sinusoidal function $\tau_z \approx -\hbar c |r|^2 \sin 2\gamma / 4\pi L$. The inset shows that the singularity at $\gamma = 0$ disappears when $|r| < 1$.

In Fig. 4 we illustrate the torque between two identical dichroic mirrors relatively rotated by $\gamma = \pi/4$ as a function of separation. Each mirror is characterized by a Lorentzian dielectric tensor with principal components

$$\epsilon_{\mu a}(\omega) = 1 + \frac{\omega_{\mu a p}^2}{\omega_{\mu a}^2 - \omega^2 - i\omega/\tau_{\mu a}}, \quad (16)$$

where $\omega_{\mu a}$ is the frequency, $\tau_{\mu a}$ the lifetime and $\omega_{\mu a p}$ the intensity of a resonance of the a -th slab corresponding to polarizing field along the μ -th principal axis. In

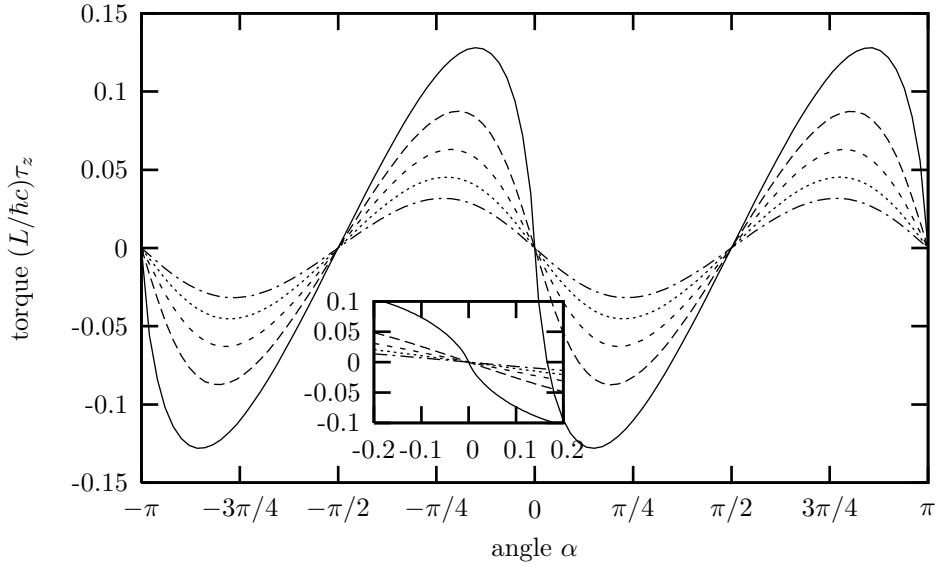


Figure 3. Torque between two lossy mirrors with reflection amplitudes $|r| = 0.6, 0.7, 0.8, 0.9, 1.0$ covered by perfect polarizers as a function of the angle γ between corresponding principal directions.

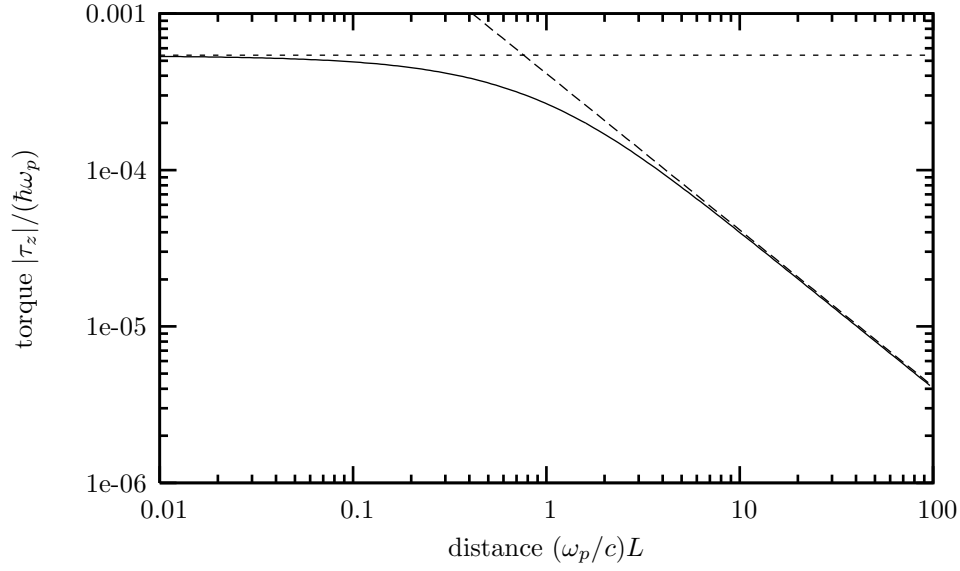


Figure 4. Torque between two identical dichroic mirrors as a function of their separation for a fixed angle $\gamma = \pi/4$ between their principal directions. The resonance parameters are $\omega_{x1p} = \omega_{y1p} = \omega_{x2p} = \omega_{y2p} = \omega_p$, $\omega_{x1} = \omega_{x2} = \omega_p$, $\omega_{y1} = \omega_{y2} = \sqrt{2}\omega_p$.

this case the characteristic frequency ω_p defines a characteristic lengthscale c/ω_p . For separations much smaller than this distance the calculation is essentially non-retarded and the 1D torque becomes constant, proportional to $\hbar\omega_p$. On the other hand, for larger separations we reach the retarded regime and the torque decays as $\hbar c/L$, as for the ideal case.

5. Conclusions

Use of the scattering approach allowed us to obtain simple expressions for the Casimir torque between anisotropic media in terms of their optical coefficients. Thus our results are applicable to arbitrary anisotropic materials and not only to semiinfinite, local, homogeneous ones. For instance, they may be readily applied to free standing or supported anisotropic films and to heterogeneous systems. We obtained analytical expressions for ideal systems which are the anisotropic counterparts to the ideal Casimir mirrors, and numerical results covering both the retarded and non-retarded regions for realistic dichroic systems with dispersive response functions. Our formalism has also permitted calculations of the torque between dissimilar materials, suggesting procedures to manipulate it, and it has been generalized to the full 3D case [18].

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